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Field-driven interface dynamics of a random soft-spin Ising model

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Abstract. The kinetics of an interface between spin-up and spin-down domains in a soft-spin Ising model with quenched random fields and driving magnetic field H is studied numerically within a discrete time dynamics at zero temperature. Spins are updated in parallel starting from a flat interface. It is found that for fields smaller than a threshold field H_c the interface is pinned while above H_c the mean velocity of the interface increases proportional to $(H - H_c)$ for two- and three-dimensional systems.

The motion of an interface in a random medium is a problem of great importance since it arises in quite different areas of research, for example the motion of a domain wall in a magnetic material with quenched disorder [1] or the immiscible-fluid displacements in porous media [2]. In magnetic systems interfaces or domain walls can be formed by cooling the system to low temperatures which results in slowly decaying metastable configurations [3]. A stable phase eventually grows out of these domains of opposite spin polarity. For large domains part of the interface can be regarded as planar. It is then an interesting problem to study the motion of such a planar structure in a random medium. Most of the attempts to solve this problem are based on equations of motion for the position of the interface which is treated as an elastic membrane [4–9]. An alternative approach is to use generalized local spin models where the interface separates two domains that correspond to regions of opposite magnetic moments in a magnetic system or different fluid phases in fluid invasion. An applied magnetic field or pressure in fluids causes one of the domains to grow. Describing the spin system by an Ising model with hard spins $S = \pm 1$ and using the conventional Monte Carlo technique one runs into problems: at very low temperatures the motion of the interface becomes extremely slow so that it is very difficult to study the asymptotic motion of the interface and its pinning behaviour. In addition, within this approach at finite temperatures true pinning can never be observed since there is always a finite probability for the interface to move. A hard-spin model in which this is avoided has been discussed by Ji and Robbins [10]. In their work the interfacial growth is simulated by increasing the driving field during simulation in such a way that the interface is not trapped. This is certainly an interesting growth model in its own right although the physical relevance of a continuous increase in the driving field remains obscure at least for magnetic systems.

The above-mentioned difficulties are not present in soft-spin models which have a non-trivial dynamics even at zero temperatures. The model considered in this paper is essentially a discrete Φ^4 theory with an assumed purely relaxational dynamics and a scalar order parameter. It should therefore be in the same universality class as the hard-spin model. Random field effects can be introduced easily. The soft-spin model has the additional advantage that it does not rule out overhangs from the very beginning as is done in models which treat interfaces as elastic membranes. Note that the model considered is similar in spirit to the one introduced and applied to domain growth in magnets by Puri *et al* [11]. As in that work one may consider the energy of the soft-spin model as the coarse-grained energy of an underlying hard-spin Ising model, a realistic model for certain magnetic systems. In this sense our model appears to be a model for magnetic systems on a mesoscopic scale.

The Hamiltonian describing the soft-spin Ising model is

$$\mathcal{H} = \sum_l \frac{u_0}{4}(S_l^2 - 1)^2 - \tfrac{1}{2}\sum_{l,l'} J_{l,l'}(S_l S_{l'} - zS_l^2\delta_{l,l'}) - \sum_l (H + B_l)S_l \tag{1}$$

where S_l are soft spins ($-\infty < S_l < \infty$) at lattice points l, the first sum with $u_0 > 0$ is a site diagonal energy fixing the length of spins (see below), the second sum is the spin interaction while the third sum is the interaction with the magnetic field. In the present paper we consider only random field effects. In this case the spin–spin interaction $J_{l,l'}$ is translationally invariant with $J_{l,l} = J$ and $J_{l,l'} = J$ where l, l' are nearest-neighbour pairs on a d-dimensional lattice and the magnetic field is split conveniently into a random field part B_l and a homogeneous driving part H as seen in equation (1). Note that in the second term in equation (1) a diagonal term is subtracted where z is the number of nearest neighbours so that the interaction term is just a discretized Laplacian. Experimentally, a realization of the random field model is a diluted antiferromagnet in an applied magnetic field [12] so that the interface dynamics studied in this paper is also relevant to these types of material.

The dynamics of the present model at zero temperature $T = 0$ is defined by the relaxation equation

$$\gamma \dot{S}_l = -\frac{\partial \mathcal{H}}{\partial S_l} \tag{2}$$

with a relaxation time proportional to γ. Finite temperature effects can be taken into account by adding an appropriate stochastic term to equation (2). In this paper, however, only $T = 0$ is considered.

To get acquainted with equation (2) two simple special cases are worth discussing. First, replacing the Hamiltonian by the first term of equation (1), one gets

$$\gamma \dot{S}_l = -u_0(S_l^2 - 1)S_l \tag{3}$$

i.e. independent spins relax to their equilibrium values $S = \pm 1$ depending on their initial conditions at a certain time. Second, replacing the Hamiltonian by the first two terms of equation (1), one gets

$$\gamma \dot{S}_l = -u_0(S_l^2 - 1)S_l + \sum_{l'} J_{l,l'} S_{l'} - zJS_l \tag{4}$$

showing that, in the homogeneous case, spin configurations with all spins up and all spins down, respectively, are stationary states of the system.

For the simulation of the motion of an interface in a random field, the equation of motion, equation (2), is integrated over a small time interval resulting in a set of difference equations

$$S_l(\tau + \Delta\tau) = S_l(\tau) + \Delta\tau \left[-u(S_l^2 - 1)S_l + \sum_{l'} S_{l'} - zS_l + h + b_l \right] \tag{5}$$

where magnetic fields are measured in units of J, time is measured in units of γ/J and $u = u_0/J$. Equation (5) is iterated starting from a flat initial interface.

To be more specific in the case of a square lattice with l_x sites in the x-direction and l_y sites in the y-direction the initial values for all spins with x-coordinates between $-l_x/2$ and 0 are set to their equilibrium value $S = 1$ while the rest of the spins are set to $S = -1$. Spins are updated in parallel. For a positive driving field the interface eventually moves to the right. After a certain number of updates (typically of the order of ten) the position x_0 of the interface is recorded and set back to $x = 0$. This is achieved by adding x_0 new rows of spins at the right-hand side and removing the same amount of rows from the left-hand side of the lattice. Then, all spin positions are relabelled. The position of the interface, x_0, is obtained from the row magnetization $\langle S \rangle_x$ which is an average over all spins in the row x. This row magnetization is a step function initially and it develops a tanh shape due to a roughening of the interface. At x_0 the row magnetization changes sign. In the x-direction the boundary conditions are $S = 1$ at the left-hand site and $S = -1$ at the right-hand site while periodic boundary conditions are used in the y-direction. Throughout, the parameter u is set equal to 0.9 and if not otherwise stated $\Delta\tau = 0.1$. The discretized version of the model, equation (5), is an interesting problem in its own right and therefore we did not change $\Delta\tau$ systematically but kept it mostly at a fixed value. The random fields b_l are independent quenched variables drawn with equal probability from an interval between $-p$ and p.

In the present investigation we are primarily interested in the averaged velocity of the interface as a function of driving field h and disorder parameter p in a steady-state situation. To reach this steady-state typically the first t_0 updates were disregarded and the following t_1 updates were taken for a calculation of the averaged velocity. Figure 1 shows the position X of the interface as a function of time t—measured in number of updates—for different values of h for $p = 0.2$. A steady state is reached after $t_0 \sim 1000$ updates. For large magnetic fields the increase in X is linear with only minor scattering. For smaller fields the fluctuations around a linear behaviour are more pronounced since then the interface is pinned in a stochastic way for some updates and then it moves again (see the inset in figure 1).

The averaged velocity V of the interface is calculated from the data points of figure 1 for $t > t_0$. Figure 2 shows V as a function of the driving field for two values of the strength p of the random field. The data are fitted with great accuracy to straight lines. For other values of p, not shown here, the same behaviour is observed. A linear dependence of V on $h - h_c$ has also been obtained by Leschhorn [7] from a mean-field calculation of an equation-of-motion approach to the interface mobility. The inset in figure 2 shows the threshold field h_c as a function of the disorder parameter p. The measuring time was only half of that used in figure 1. Although there is some scattering in the data we believe that it is safe to say that pinning occurs for any amount of disorder if the driving field is small enough. The results discussed above were obtained for square lattices with $l_x = 180$ and $l_y = 300$. Finite-size effects have been checked but they appear to be negligible.

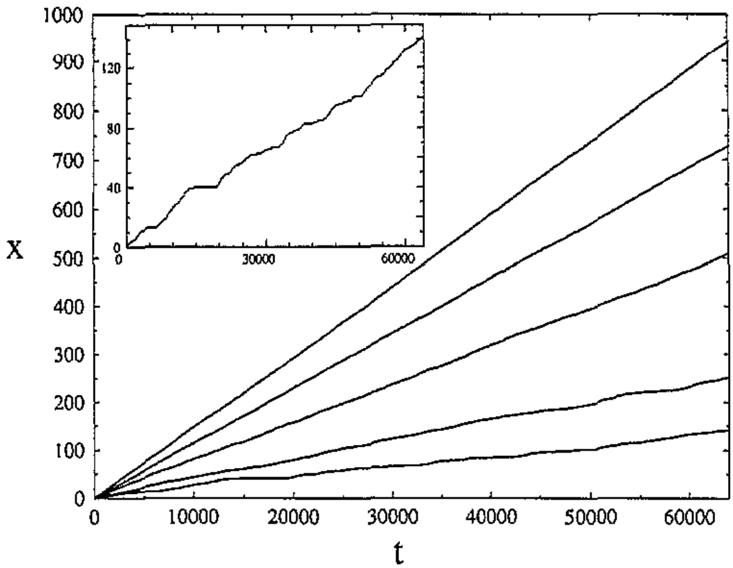

Figure 1. Position X of the interface in units of lattice constant against time t (number of updates) for different values of driving field h. $h = 0.05$, 0.04, 0.03, 0.02, 0.016 25 (from above). The inset shows the curve $h = 0.016\,25$ enlarged. $p = 0.2$ for all curves.

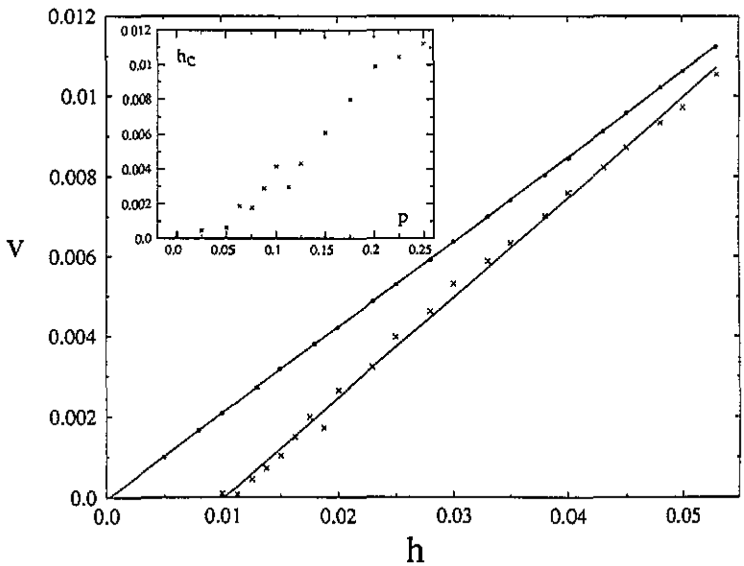

Figure 2. Interface velocity V in units of lattice constant/update against driving field h in $d = 2$: upper curve, $p = 0$; lower curve, $p = 0.2$. h and p are dimensionless as explained in the text. The inset shows the threshold field h_c against p.

Some results for three-dimensional systems are displayed in figure 3. The calculations were restricted to $p = 0$ and $p = 0.2$ and the system size was $90 \times 60 \times 60$.

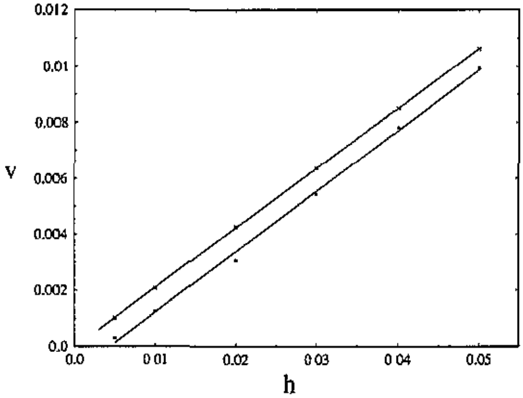

Figure 3. Interface velocity V against driving field h in $d = 3$ for $p = 0$ (upper curve) and $p = 0.2$ (lower curve).

Obviously, the linear size of the lattice had to be smaller than that for two-dimensional systems but our experience from the calculations in two dimensions was that the size of the system is not crucial in determining the critical fields and the corresponding exponent, so we consider this lattice to be sufficiently large. Perhaps surprisingly, figure 3 shows that in three dimensions a linear dependence of V on $h - h_c$ is also observed.

Finally, we have varied $\Delta\tau$ in equation (5) to study the effect of time discretization. Decreasing $\Delta\tau$ by a factor of five again a linear dependence of V on $h - h_c$, is observed with practically no change in the threshold field but with some change in the slope of that line. This result was obtained for $p = 0.2$. Since there appears to be no dramatic effect on time discretization we did not dwell any further on this problem.

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References

[1] Komura S and Furukawa H 1988 *Dynamics of Ordering Processes in Condensed Matter* (New York: Plenum)
[2] Scheidegger A E 1984 *The Physics of Flow in Porous Media* (Toronto: University of Toronto Press)
[3] Nattermann T and Rujan P 1989 *Int. J. Mod. Phys.* B **3** 1597
[4] Bruinsma R and Aeppli G 1984 *Phys. Rev. Lett.* **52** 1547
[5] Koplik J and Levine H 1985 *Phys. Rev.* B **32** 280
[6] Feigelman M V 1983 *Sov. Phys.-JETP* **58** 1076
[7] Leschhorn H 1992 *J. Phys. A: Math. Gen.* **25** L555
[8] Nattermann T, Stepanow S, Tang L-H and Leschhorn H *J. Physique* to be published
[9] Roland C and Hong Guo 1992 *J. Phys. A: Math. Gen.* **25** 3507
[10] Ji H and Robbins M 1991 *Phys. Rev.* A **44** 2538
[11] Puri S, Chowdhury D and Parekh N 1991 *J. Phys. A: Math. Gen.* **24** L1087
Puri S and Parekh N 1992 *J. Phys. A: Math. Gen.* **25** 4127
[12] Fishman S and Aharony A 1979 *J. Phys. C: Solid State Phys.* **12** L729